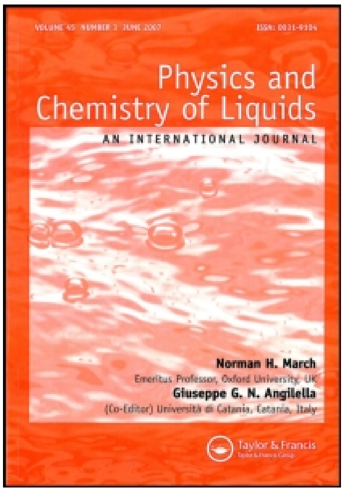

Physics and Chemistry of Liquids

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713646857

Polymer growth in solution and the dynamic epidemic model: dimensionality and critical exponents

Z. D. Zhang[a]; N. H. March[bc]
[a] Shenyang National Laboratory for Materials Science, Institute of Metal Research and International Centre for Materials Physics, Chinese Academy of Sciences, Shenyang 110016, P.R. China [b] Department of Physics, University of Antwerp, Antwerp, Belgium [c] Oxford University, Oxford, UK

Online publication date: 03 June 2010

To cite this Article Zhang, Z. D. and March, N. H.(2010) 'Polymer growth in solution and the dynamic epidemic model: dimensionality and critical exponents', Physics and Chemistry of Liquids, 48: 3, 403 — 408
To link to this Article: DOI: 10.1080/00319100903539538
URL: http://dx.doi.org/10.1080/00319100903539538

Physics and Chemistry of Liquids
Vol. 48, No. 3, June 2010, 403–408

Polymer growth in solution and the dynamic epidemic model: dimensionality and critical exponents

Z.D. Zhang[a]* and N.H. March[bc]

[a]*Shenyang National Laboratory for Materials Science, Institute of Metal Research and International Centre for Materials Physics, Chinese Academy of Sciences, 72 Wenhua Road, Shenyang 110016, P.R. China;* [b]*Department of Physics, University of Antwerp, Antwerp, Belgium;* [c]*Oxford University, Oxford, UK*

(*Received 26 November 2009; final version received 8 December 2009*)

Recent interest in solutions of the static and dynamic epidemic models has led us to reconsider some features of the critical exponents of these models. In particular, our starting point is that the known values of such exponents take different values depending on dimensionality *d*, for dynamic epidemics, depending also on the looped nature of the chain, but the universality is regained for $d > 1$. According to this superuniversality property for $d > 1$, we could predict that the critical exponents γ and ν for dynamic epidemics on a square lattice should be 43/18 and 4/3, respectively. Furthermore, from numerical results in literature, we predict the critical exponents for the Potts model in three dimensions and the percolation exponents ($q = 1$) for $d = 1$, 2, 3, 4, 5 and $d \geq 6$. Other areas of relevance beyond that in the title embrace conduction in heterogeneous media as well as a description of the spreading of a fluid in a medium possessing mobile impurities.

Keywords: polymer growth; solutions; dynamic and static epidemics; critical exponents

There has been considerable interest over the past decade or so in the so-called dynamic epidemic model [1,2]. While our specific interest is expressed in the title of this article, as concerning specifically the polymer growth in solution, we refer in our concluding comments to two other relevant areas which are also of current interest to our own studies. Our prime focus further is that of critical exponents, following the conjectured solution by one of us [3] on the 3D Ising model, to which we return briefly below.

As a little further background, it is important to note that Vandewalle and Ausloos [1] presented an exact solution of such a dynamic epidemic model referred to above, using the 'growth–transfer-matrix' method for a variety of chains and trees containing loops. In the subsequent work of Ivanova [2], the dynamic epidemic model has been solved exactly on chains and a Bethe tree, all of which are decorated with either consecutive hexagon or tetrahedron loops. Earlier background material can be consulted in the review by Isichenko [4] which covers percolation, statistical topography as well as transport in random media. In the context of percolation, it is

*Corresponding author. Email: zdzhang@imr.ac.cn

ISSN 0031–9104 print/ISSN 1029–0451 online

DOI: 10.1080/00319100903539538
http://www.informaworld.com

noteworthy that, from table III of Isichenko's [4] review, percolation critical exponents are compared for $d=2$ and $d=3$. From that table, it can be seen that such percolation exponents for $d=2$ lattices numerically vary in certain ranges: $\beta=0.138$–0.15, $\gamma=2.38-2.43$, $\nu=1.33$–1.35, $\mu=1.10$–1.32, etc. However, the exact values are given in [4]: $\alpha=-2/3$, $\beta=5/36$, $\gamma=43/18$, $\delta=91/5$, $\nu=4/3$, $\eta=5/24$, $\mu=1.25$, $\tau=(2\nu d-\beta)/(\nu d-\beta)=187/91$ and $d_c=d-\beta/\nu=91/48$. It is to be noted at this point that the exact values for 2D are derived from conjectured results for the extended Potts model and magnetic eigenvalue is given by Pearson [5]. If one put $x=-2/3$ into Equation (7) of Pearson's [5] article for the critical exponents, one can reproduce the exact values given above.

The exact values of the critical exponents for 2D Potts model were conjectured independently by Pearson [5] and Nienhuis *et al.* [6]. These predicted critical exponents are supported by the exact values of Ising model ($q=2$) [7], hard hexagon lattice gas ($q=3$) [8,9] and Baxter–Wu model ($q=4$) [10–13]. These exact results led firm support to the correctness of the conjectures, which are compared with various numerical means in table VI of Wu's [14] review. The Potts model with $q=1$ corresponds to the percolation model, according to equation (2) of Pearson's article [5], one has $x=-2/3$ for the critical side, while $q=2$ is for an Ising model with $x=-1/2$ on the critical side. The solution of $x=2/3$ or $1/2$ corresponds to multicritical side for $q=1$ or 2. In table V of Wu's [14] review, critical exponents conjectured for the 2D Potts model were tabulated, which are shown in this article as Table 1. Note that the clerical errors (y_h for $q=1$ and η for $q=4$) in Wu's [14] Table V have been corrected. According to figure 2 in [14] and also figure 1 in [15], there is a critical value q_c (d) beyond which the transition is mean-field like, while it is of first order for $q>2$ and second order for $q\leq 2$. For two dimensions (2Ds), the transition becomes the first order for $q>4$, making the values for $q=\infty$ in Table 1 inappropriate. Nevertheless, for a complete comparison, we still list those values obtained by extrapolation of the data in table I of den Nijs' article [16], and by assuming a continuous transition for $q=\infty$.

As to the percolation critical exponents for dimensionality $d=3$, Isichenko lists in table III of [4] the values $\alpha=-\ 0.64$, $\beta=0.39-0.454$, $\gamma=1.63-1.91$, $\delta=4.81$, $\nu=0.82-0.94$, $\mu=1.4-2.46$, $\tau=2.19$ and $d_c=2.484$. Therefore, we can predict that the critical exponents for dynamic epidemics on 3D lattices should be close to the numerical results quoted in Isichenko's [4] review. Furthermore, in table 4.2 of [17], Hansen quotes critical exponents for percolation in 2D as already given in this article, while those in 3D are $\beta=0.44$, $\gamma=1.76$, $\nu=0.88$, $\tau=2.2$ and $d_c=2.5$. These are indeed very close to the values tabulated by Isichenko [4].

Table 1. Critical exponents for the Potts model in 2D.

q	y_t	y_h	α	β	γ	δ	η	ν
0	0	2	$-\infty$	1/6	∞	∞	0	∞
1	3/4	91/48	$-2/3$	5/36	43/18	91/5	5/24	4/3
2	1	15/8	0	1/8	7/4	15	1/4	1
3	6/5	28/15	1/3	1/9	13/9	14	4/15	5/6
4	3/2	15/8	2/3	1/12	7/6	15	1/4	2/3
∞	2	2	1	0	1	∞	0	1/2

It is to be understood that because the looped character of 2D and 3D lattices is not an important factor for the critical exponents corresponding to the dynamic epidemic model, and also because of the superuniversality property for $d > 1$, the results for percolation critical exponents in other physical systems can be utilised for understanding the critical behaviour of the dynamic epidemics. It would be of considerable interest if one could find the exact solution for percolation critical exponents on the 3D lattices. However, it is more difficult to construct the relation for critical exponents of the 3D Potts model, since a first-order phase transition may exist in 3D with $q \geq 3$. In figure 2 of [14], the black circle indicates the assumed first-order transition for $d=3$, $q=3$. Nienhuis *et al.* [18] studied q-state Potts model in general dimension and concluded from calculations of renormalisation group for dimensions 1.58, 2 and 2.32 that the three-state Potts model in 3D undergoes a first-order phase transition. However, there has been some argument predicting a continuous transition for $q=3$, $d=3$ (for instance, [19]). It is thought that even the transition for $d=3$, $q=3$ is of first order, which is very weak (close to the border) so that it could be treated as a second-order phase transition for understanding the critical behaviour. From the numerical results in table 1 of [19] and table III of [4], one can predict a set of critical exponents for $d=3$, $q=3$, under the assumption of the existence of a continuous transition. On the other hand, in consideration of the numerical results for $d=3$, $q=1$ [17], one can also predict the critical exponents for $d=3$, $q=1$. The results are predicted based on the following: (1) we assume that the thermal exponent, y_t, and the magnetic exponent, y_h, are related to dimensions, because in 2D, $y_t=0$, $y_h=2$ for $q=0$, while $y_t=2$, $y_h=2$ for $q=\infty$. Thus, we have $y_t=0$, $y_h=3$ for $q=0$, while $y_t=3$, $y_h=3$ for $q=\infty$ for the 3D Potts model. In the latter case, we disregard the existence of the first-order phase transition for $q=\infty$. (2) We assume that all the critical exponents for the 3D Potts model are simple fractions or integers. From the numerical results in table 1 of [19], one immediately obtains $\alpha=1/2$, $\beta=1/4$ $\gamma=1$ and $\delta=5$ for $q=3$. From table 4.2 of [17], one obtains $\beta=11/24$, $\gamma=7/4$, $\nu=8/9$ for $q=1$. These critical exponents and others obtained by the scaling laws are consistent with the numerical data in table III of [4] and also in table III of [20].

These predicted results are listed in Table 2 for the 3D Potts model, together with the conjectured solution of ZDZ [3] for the 3D Ising model ($d=3$, $q=2$). Here $\beta=1/2$ for $d=3$, $q=0$ is derived by extrapolation, since it cannot be obtained directly from the scaling laws. Comparing the values in Tables 1 and 2, we find the following laws: (1) for both 2D and 3D, the thermal exponent y_t increases from 0 for $q=0$ to the value d for $q=\infty$, while the magnetic exponent y_h decreases from the

Table 2. Critical exponents for the Potts model in 3D.

q	y_t	y_h	α	β	γ	δ	η	ν
0	0	3	$-\infty$	1/2	∞	∞	-1	∞
1	9/8	159/64	$-2/3$	11/24	7/4	53/11	1/32	8/9
2	3/2	39/16	0	3/8	5/4	13/3	1/8	2/3
3	2	5/2	1/2	1/4	1	5	0	1/2
∞	3	3	1	0	1	∞	-1	1/3

value d for $q=0$ to a minimum and then increases back to the value d for $q=\infty$. (2) The critical exponents α for the Potts models in 2D and 3D are exactly the same for the same q value if $q \leq 2$. As q varies from 0 to ∞, the critical exponent α increases from $-\infty$ to 1. (3) With increasing q from 0 to ∞, the critical exponent β decreases from 1/6 (or 1/2) to 0 for 2D (or 3D). Meanwhile, the critical exponent γ decreases from ∞ to 1 for both 2D and 3D, the critical exponent δ decreases from infinite to a minimum and then turns back to infinite, the critical exponent η increases from 0 for 2D (or -1 for 3D) to a maximum and then decreases to 0 (or -1 for 3D), the critical exponent ν decreases from infinite to the value $1/d$ for both 2D and 3D. Of course, it should be emphasised that due to the existence of the first-order phase transition, the values for $q=\infty$ in Table 1 and for $q=3$ and ∞ in Table 2 are listed only for better understanding the tendency of variation of the critical exponents. Because there is a superuniversality for $d>1$ for the critical exponents, we suggest that the values in Table 2 for $d=3$, $q=1$ (i.e. the 3D percolation model) can be used for studying the critical behaviour of static and dynamic epidemics in 3D lattices.

The percolation exponents listed in table II of [21] are important for understanding, at a deeper level, the critical behaviours of the $q=1$ Potts model for different dimensions. From that table, we predict all the percolation exponents for $d=2$, 3, 4, 5 and $d \geq 6$, also based on an assumption that all the percolation exponents for the Potts model are simple fractions or integers. The predicted results are listed in Table 3 for the percolation exponents for $d=1$, 2, 3, 4, 5 and $d \geq 6$. These values agree with Appendix 1, table of critical exponents, in [22] and also the mean-field values obtained exactly for a Bethe lattice by Reich and Leath [23] and figure 8 of [24] (and Ref. [43] therein, i.e. [25]). The values in Table 3 for $q=1$, $d=2$ and 3 are already given in Tables 1 and 2, respectively. It is interesting to note that the percolation exponent α is equal to $-2/3$ for $1 < d \leq 4$, while the percolation exponent η equals 0 for $d \geq 4$. The percolation exponent ν can be written as 48/36, 32/36, 24/36, 20/36 and 18/36 for $d=2$, 3, 4, 5 and $d \geq 6$, respectively. The differences between the percolation exponents ν of these neighbouring dimensions are 16/36, 8/36, 4/36 and 2/36, which form a geometric series. From this fact, we can derive the relations for the percolation exponents ν: $3\nu_{3d}-\nu_{2d}-2\nu_{4d}=0$, $3\nu_{4d}-\nu_{3d}-2\nu_{5\mathrm{d}}=0$, $3\nu_{5\mathrm{d}}-\nu_{4\mathrm{d}}-2\nu_{6\mathrm{d}}=0$. But, the percolation exponent $\nu=1/2$ for $d \geq 6$ disagrees with static exponent $\nu=1$ for Bethe lattice ($d=\infty$) of [1,2]. According to [15,16], there are three known exact points $(q,d)=(1,6)$, $(2,4)$, $(4,2)$. For $q=1$ and $q>6$, the percolation exponents should take the same values as $(q,d)=(1,6)$. The percolation exponents

Table 3. Percolation exponents ($q=1$) for $d=1$, 2, 3, 4, 5 and $d \geq 6$.

d	y_t	y_h	α	β	γ	δ	η	ν
1	1	1	1	0	1	∞	1	1
2	3/4	91/48	−2/3	5/36	43/18	91/5	5/24	4/3
3	9/8	159/64	−2/3	11/24	7/4	53/11	1/32	8/9
4	3/2	3	−2/3	2/3	4/3	3	0	2/3
5	9/5	7/2	−7/9	5/6	10/9	7/3	0	5/9
6	2	4	−1	1	1	2	0	1/2

for high dimensions should be the same as the mean-field (Bethe lattice) values [21–25], as proved in [26].

We wish to conclude this article with three fairly brief sections, the first of which will be concerned with the influence of loops. In this context, we note that Ivanova [2] studied static and dynamic epidemic models on chains and trees with 2D and 3D loops. From table 1 of [2], the critical exponents for static epidemics on chains are $\nu = \gamma = 1$, but those for the dynamic epidemics are $\nu = \gamma = 3$, which agree with the findings of Vandewalle and Ausloos [1]. This clearly shows that for $d = 1$, the critical exponents for static epidemics differ from those of the dynamic epidemic model, and that the chains with squares, triangles, hexagons and tetrahedrons have the same critical exponents. For static epidemics on a tree with tetrahedrons, the critical exponents have the same values as those given by Vandewalle and Ausloos [1] (tree with squares and triangles): $\nu = \gamma = 1$ for both static and dynamic epidemic models. The major conclusion here is that universality is lacking for the dynamic epidemic model on a 1D lattice, but on trees the critical exponents are unaffected by the introduction of a dynamic interaction and loops, and there is a superuniversal for $d > 1$.

Our second concluding comment concerns the critical exponents for the 2D and 3D Potts models for different states q and also the percolation exponents ($q = 1$) for $d = 1$, 2, 3, 4, 5 and $d \geq 6$. From the numerical results in literature, we predict the critical exponents for the 3D Potts models with different q and also the percolation exponents ($q = 1$) for different dimensions. As to future studies, we believe it would be of considerable interest if one could understand the discrepancy between the mean-field percolation exponent $\nu = 1/2$ for $d \geq 6$ [21–25] and static exponent $\nu = 1$ for the Bethe lattice ($d = \infty$) of [1,2].

The very brief concluding comment of this article concerns the relevance to other areas than polymer growth in solution stressed in the title. These also embrace electrical conduction in heterogeneous media, as well as describing the spreading of a fluid in a medium possessing mobile impurities.

Acknowledgements

Z.D. Zhang acknowledges the support of the National Natural Science Foundation of China (Grant numbers 50831006 and 10674139). N.H. March wishes to acknowledge his continuing affiliation with the University of Antwerp (UA). Specifically, he thanks Professors D. Lamoen and C. Van Alsenoy for partial financial support from UA via BOF-NOI.

References

[1] N. Vandewalle and M. Ausloos, Phys. Rev. **E54**, 3499 (1996).
[2] K. Ivanova, Phys. Rev. **E57**, 4827 (1998).
[3] Z.D. Zhang, Phil. Mag. **87**, 5309 (2007).
[4] M.B. Isichenko, Rev. Mod. Phys. **64**, 961 (1992).
[5] R.B. Pearson, Phys. Rev. **B22**, 2579 (1980).
[6] B. Nienhuis, E.K. Riedel, and M. Schick, J. Phys. **A13**, L189 (1980).
[7] L. Onsager, Phys. Rev. **65**, 117 (1944).
[8] S. Alexander, Phys. Lett. **A54**, 353 (1975).
[9] R.J. Baxter, J. Phys. **A13**, L61 (1980).

[10] I.G. Enting, J. Phys. **A8**, L35 (1975).
[11] E. Domany and E.K. Riedel, J. Appl. Phys. **49**, 1315 (1978).
[12] R.J. Baxter and F.Y. Wu, Phys. Rev. Lett. **31**, 1294 (1973).
[13] R.J. Baxter, M.F. Sykes, and M.G. Watt, J. Phys. **A8**, 245 (1975).
[14] F.Y. Wu, Rev. Mod. Phys. **54**, 235 (1982).
[15] F.Y. Wu, J. Appl. Phys. **56**, 2421 (1984).
[16] M.P.M den Nijs, Physica **A95**, 449 (1979).
[17] A. Hansen, in *Statistical Models for the Fracture of Disorder Media*, edited by H.J. Herrmann and S. Roux (Elsevier Science Publisher, Amsterdam, 1990), Chap. 4, pp. 115–158.
[18] B. Nienhuis, E.K. Riedel, and M. Schick, Phys. Rev. **B23**, 6055 (1981).
[19] T.W. Burkhardt, H.J.F. Knops, and M. den Nijs, J. Phys. **A9**, L179 (1976).
[20] M. Sahimi, Rev. Mod. Phys. **65**, 1393 (1993).
[21] T. Nakayama, K. Yakubo, and R.L. Orbach, Rev. Mod. Phys. **66**, 381 (1994).
[22] J.W. Essam, Rep. Prog. Phys. **43**, 833 (1980).
[23] G.R. Reich and P.L. Leath, J. Stat. Phys. **19**, 611 (1978).
[24] C.W. Nan, Progr. Mater. Sci. **37**, 1 (1993).
[25] R. Zallen, *The Physics of Amorphous Solids* (Wiley, New York, 1983).
[26] T. Hara and G. Slade, Commun. Math. Phys. **128**, 333 (1990).